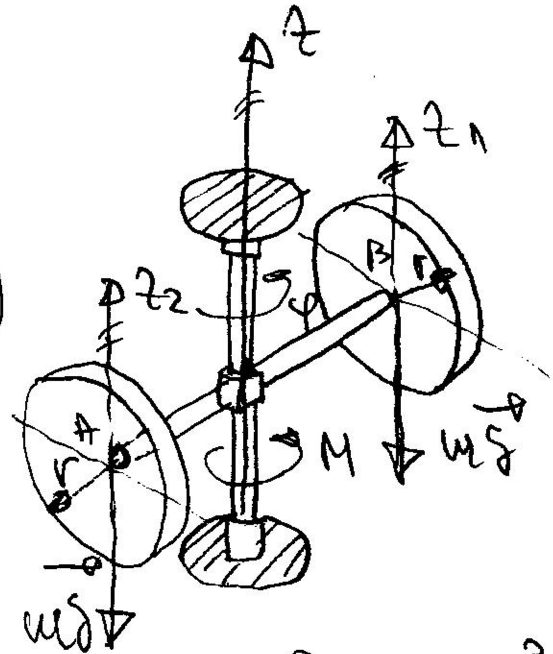
8.11 $2a$

$m,\ r$

$M$

---

$\varphi - ?$




$J_{z1} = \frac{m}{4} r^2$

$$\frac{dL_{oz}}{dt} = \sum_i M_{oz}(\vec{F}_i)$$

$$L_{oz} = J_{oz}\dot{\varphi}$$

$$J_{oz} = 2\left(\frac{m}{4}r^2 + ma^2\right) = \frac{1}{2}m\left(r^2 + 4a^2\right)$$

$$L_{oz} = \frac{m}{2}\left(r^2 + 4a^2\right)\dot{\varphi}$$

$$\frac{dL_{oz}}{dt} = \frac{m}{2}\left(r^2 + 4a^2\right)\ddot{\varphi}$$

$$\sum_i M_{oz}(\vec{F}_i) = M$$

$$\frac{m}{2}\left(r^2 + 4a^2\right)\ddot{\varphi} = M$$

$$\ddot{\varphi} = \frac{2M}{m(r^2 + 4a^2)} = \frac{d\dot{\varphi}}{dt} \Big|\cdot dt \Big|\cdot\int$$

$$\dot{\varphi} = \dot{\varphi}_0 + \frac{2M}{m(r^2+4a^2)}t = \frac{d\varphi}{dt}\Big| dt \Big|\cdot\int$$

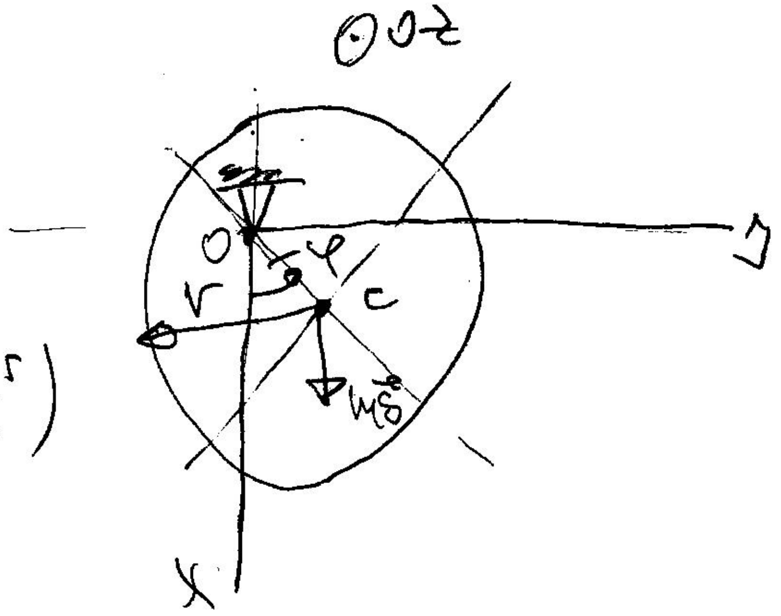
$$\varphi(t) = \varphi_0 + \dot{\varphi}_0 t + \frac{Mt^2}{m(r^2+4a^2)}$$

8.22

$r$

$r/2$

$T - ?$




$$\frac{dL_{Oz}}{dt} = \sum_i M_{Oz}(\vec{F}_i^{e})$$

$$L_{Oz} = J_{Oz}\dot{\varphi}$$

$$J_{Oz} = \frac{m}{2}r^2 + m\left(\frac{r}{2}\right)^2 = \frac{mr^2}{2} + \frac{mr^2}{4} = \frac{3mr^2}{4}$$

$$L_{Oz} = \frac{3mr^2}{4}\dot{\varphi}$$

$$\frac{dL_{Oz}}{dt} = \frac{3mr^2}{4}\ddot{\varphi}$$

$$\sum_i M_{Oz}(\vec{F}_i^{e}) = -mg\frac{r}{2}\sin\varphi$$

МАЛЫЕ ОСЦИЛЛЯЦИИ

$$\sin\varphi \approx 0 + 1\cdot\varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} + \ldots$$

$$\boxed{\sin\varphi \approx \varphi}$$

$$\frac{3\cancel{m}r^{\cancel{2}}}{\cancel{4}_2}\ddot{\varphi} = -\cancel{m}g\frac{\cancel{r}}{\cancel{2}}\varphi$$

$$\frac{3r}{2}\ddot{\varphi} = -g\varphi \Rightarrow \ddot{\varphi} + \frac{2g}{3r}\varphi = 0, \quad \ddot{\varphi} + \omega^2\varphi = 0$$

$$\omega^2 = \frac{2g}{3r}, \quad T = \frac{2\pi}{\omega} = \pi\sqrt{\frac{6r}{g}}$$

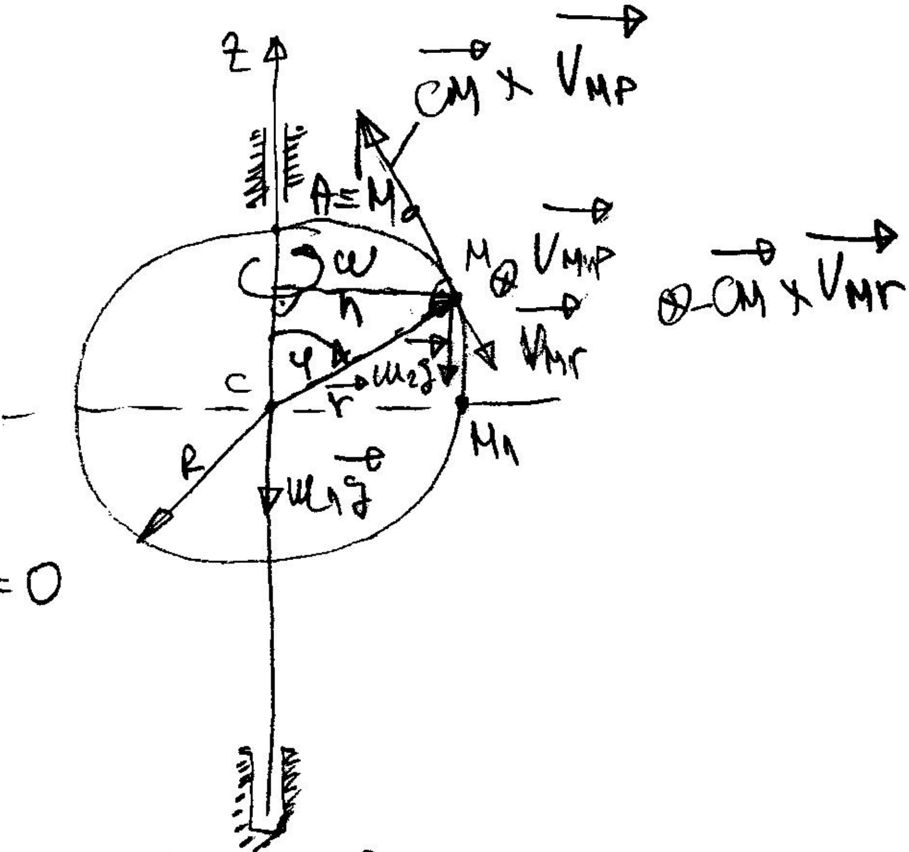
8.24 $m_1$

$R$

$\omega_0$

$m_2$

$\omega_1^d - ?$




$$\frac{dL_{cz}}{dt} = \sum_{i=1}^{n} M_{cz}(\vec{F}_i^s) = 0$$

$$L_{cz} = \text{const.}$$

$$t_0 = 0: \quad L^d_{cz}(t_0) = J_{cz}\omega_0 = \frac{1}{4} m_1 R^2 \omega_0$$

$$L^M_{cz}(t_0) = 0$$

$$t: \quad L^d_{cz}(t) = J^d_{cz}\omega = \frac{1}{4} m_1 R^2 \omega$$

$$L^M_{cz}(t) = (\vec{r}_M \times m_2 \vec{V}_M)\cdot\vec{k} = [m_2 \vec{CM} \times (\vec{V}_{Mp} + \vec{V}_{Mr})]\cdot\vec{k} =$$

$$= [m_2 \vec{CM} \times \vec{V}_{Mp} + m_2 \vec{CM} \times \vec{V}_{Mr}]\cdot\vec{k} =$$

$$= m_2(\vec{CM} \times \vec{V}_{Mp})\cdot\vec{k} + m_2 \underbrace{(\vec{CM} \times \vec{V}_{Mr})\cdot\vec{k}}_{0} =$$

$$= m_2 \overline{CM}\, V_{Mp} \cdot 1 \cdot \cos(90^\circ - \varphi) = m_2 \overline{CM} \sin\varphi\, V_{Mp} =$$

$$= m_2 \underbrace{\overline{CM} \sin\varphi}_{h} h\omega = m_2 h^2 \omega$$

$$L_{cz}(t) = \frac{1}{4} m_1 R^2 \omega + m_2 h^2 \omega$$

$$t = t_1 \quad h = R, \quad L_{cz}(t_1) = \frac{1}{4} m_1 R^2 \omega_1 + m_2 R^2 \omega_1$$

$$\frac{1}{4} m_1 R^2 \omega_1 + m_2 R^2 \omega_1 = \frac{1}{4} m_1 R^2 \omega_0 \quad / : R^2$$

$$\omega_1 \left( \frac{1}{4} m_1 + m_2 \right) = \frac{1}{4} m_1 \omega_0$$

$$\boxed{\omega_1} = \frac{\frac{1}{4} m_1 \omega_0}{\frac{1}{4} m_1 + m_2} = \boxed{\frac{m_1 \omega_0}{m_1 + 4 m_2}}$$

8.25 $m_1$

$r$

$\omega_0$

$V^M_{r0} = 0$

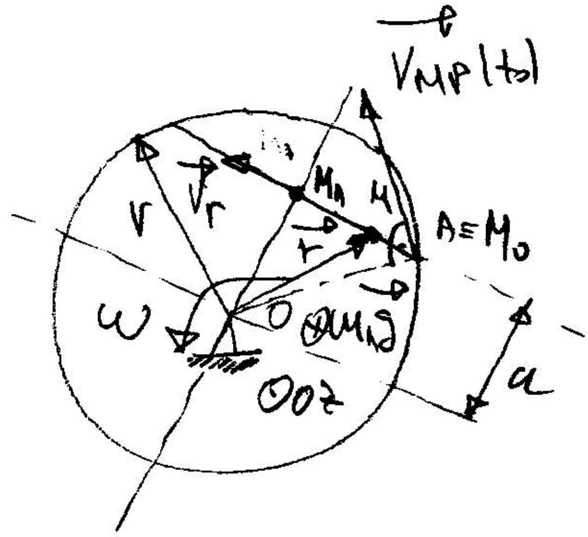
$M_0 \equiv A$

$m_2$

$V_r$

---

$\omega_1 - ?$




$$\frac{dL_{Oz}}{dt} = \sum_{i=1}^{n} M_{Oz}(\vec{F}_i^e) = 0 \qquad L_{Oz} = const.$$

$$t_0 = 0: \quad L^d_{Cz}(t_0) = J^d_{Cz} \omega_0 = \frac{1}{2} m_1 r^2 \omega_0$$

$$L^M_{Oz}(t_0) = (\vec{OA} \times m_2 \vec{V}_{M_0}(t_0)) \cdot \vec{k} =$$

$$= m_2 (\vec{OA} \times (\vec{V}_{Mr}(t_0) + \vec{V}_{Mp}(t_0))) \cdot \vec{k} =$$

$$= m_2 (\underbrace{\vec{OA} \times \vec{V}_{Mr}(t_0)}_{0} + \vec{OA} \times \vec{V}_{Mp}(t_0)) \cdot \vec{k} =$$

$$= m_2 (\vec{OA} \times \vec{V}_{Mp}(t_0)) \cdot \vec{k} = m_2 \overline{OA} \cdot \overline{OA} \omega_0 \cdot 1 \cdot 1 =$$

$$= m_2 \overline{OA}^2 \omega_0 = m_2 r^2 \omega_0$$

$$L_{Cz}(t_0) = \frac{1}{2} m_1 r^2 \omega_0 + m_2 r^2 \omega_0 = \frac{1}{2}\omega_0 (m_1 + 2m_2) r^2$$

---

$$t = t_1 \qquad L^{d}_{Cz}(t_1) = J^{d}_{Cz}\,\omega_1 = \frac{1}{2} m_1 a^2 \omega_1$$

$$L^{M}_{Cz}(t_1) = m_2 a^2 \omega_1 + m_2 (\vec{OM_1} \times \vec{V_r}) \cdot \vec{K} = m_2 a^2 \omega_1 + m_2 a V_r$$

$$L_{Cz}(t_1) = \frac{1}{2} m_1 r^2 \omega_1 + m_2 a^2 \omega_1 + m_2 a V_r$$

---

$$\frac{1}{2} m_1 r^2 \omega_0 + m_2 r^2 \omega_0 = \frac{1}{2} m_1 r^2 \omega_1 + m_2 a^2 \omega_1 + m_2 a V_r \,| \cdot 2$$

$$\omega_0 (m_1 + 2m_2) r^2 = \omega_1 (m_1 r^2 + 2m_2 a^2) + 2m_2 a V_r$$

$$\omega_1 = \frac{\omega_0 (m_1 + 2m_2) r^2 - 2m_2 a V_r}{(m_1 r^2 + 2m_2 a^2)}$$

---

8.31 $Q$

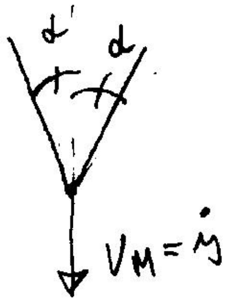
$P$

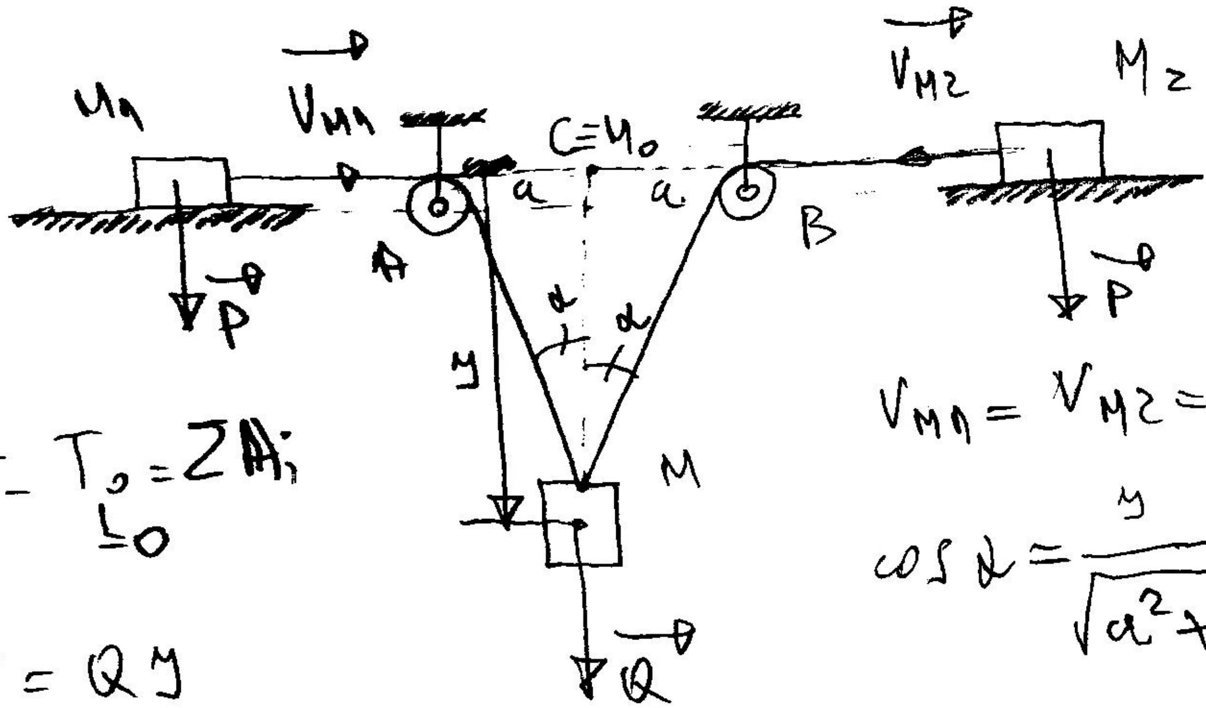
$t_0 = 0 \quad M_0 \equiv C$

$a \quad , V_0 = 0$

---

$V_M(y) - ?$




$V_{M1} = V_{M2} = \dot{y} \cos \alpha$

$\cos \alpha = \dfrac{y}{\sqrt{a^2 + y^2}}$

$T - \underbrace{T_0}_{0} = \sum A_i$

$T = Q y$

$T = T_{M1} + T_{M2} + T_M$

$T_{M1} = \dfrac{1}{2} \dfrac{P}{g} V_{M1}^2 = \dfrac{1}{2} \dfrac{P}{g} V_M^2 \cos^2 \alpha = T_{M2}$

$T_M = \dfrac{1}{2} \dfrac{Q}{g} V_M^2$

---

$\dfrac{1}{2} \dfrac{Q}{g} V_M^2 + \dfrac{P}{g} V_M^2 \cos^2 \alpha = Q y$

$\dfrac{V_M^2}{2g} \left( Q + 2P \dfrac{y^2}{a^2 + y^2} \right) = Q y \Rightarrow$

$V_M = \sqrt{\dfrac{2 g Q y}{Q + \dfrac{2y^2}{a^2 + y^2} P}}$

8.35

$m_1 = 4m$

$R$

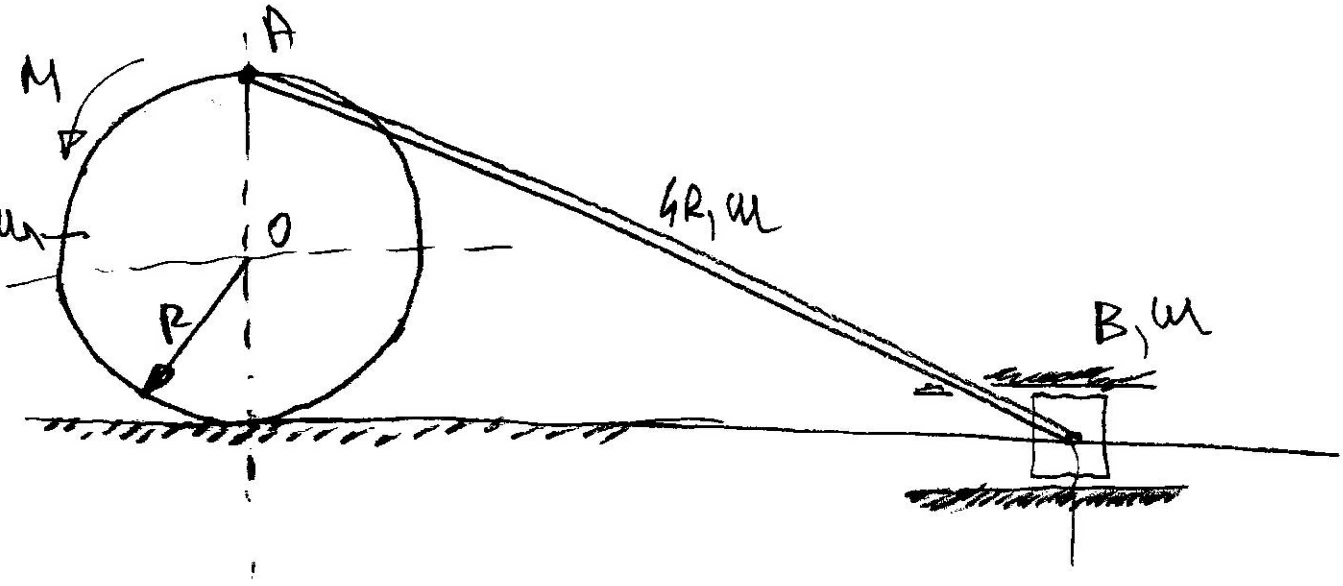
$M = 4mgR$

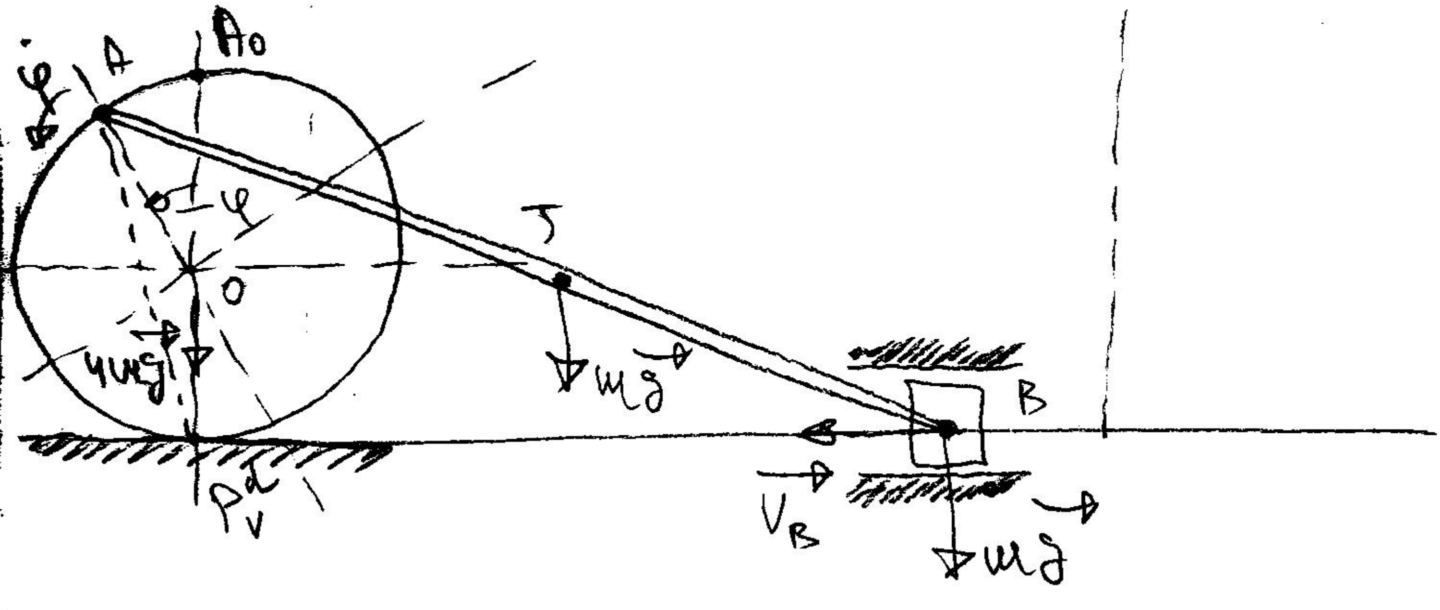
$\overline{AB} = 4R,\ m$

$B : m$

$\varphi_1 = \frac{\pi}{4}$

$W_d - ?$



$$T - T_0 = \sum_{i=1}^{n} A_i$$
$$\llcorner 0$$

$$T = T_d + T_{AB} + T_B$$

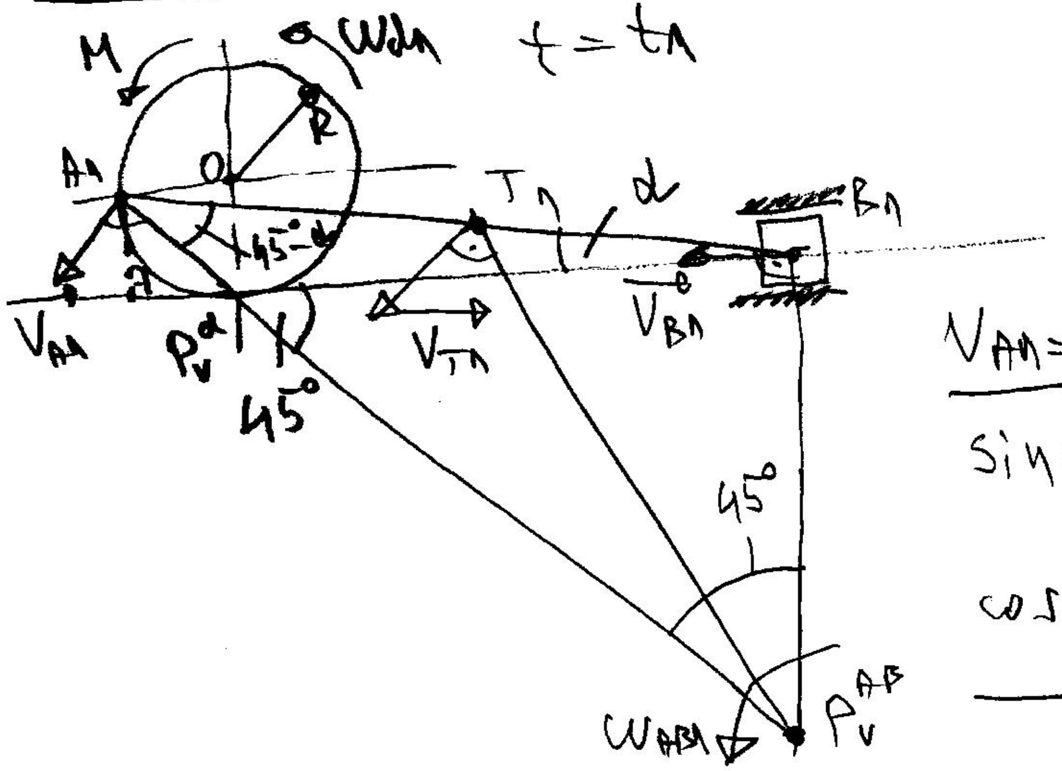
$$T_d = \frac{1}{2} J_0 \dot{\varphi}^2 + \frac{1}{2} 4m V_0^2 \quad , \quad V_0 = R\dot{\varphi}$$

$$T_d = 2mR^2\dot{\varphi}^2 + \frac{1}{\cancel{2}}\frac{1}{\cancel{2}} 4mR^2\dot{\varphi}^2 = 3mR^2\dot{\varphi}^2$$

$$T_{AB} = \frac{1}{2} m V_T^2 + \frac{1}{2} J_T \omega_{AB}^2$$

$$T_B = \frac{1}{2} m V_B^2$$

---




$$V_{A1} = R\sqrt{2}\,\omega_{d1}$$

$$\sin\alpha = \frac{R}{4R} = \frac{1}{4}$$

$$\cos\alpha = \frac{\sqrt{15}}{4}$$

$$\frac{4R}{\sin 45^\circ} = \frac{\overline{B_1P_V^{AB}}}{\sin(45^\circ - \alpha)} = \frac{\overline{A_1P_V^{AB}}}{\sin(90^\circ + \alpha)}$$

$$\frac{4R}{\frac{\sqrt{2}}{2}} = \frac{\overline{B_1P_V^{AB}}}{\sin(45^\circ - \alpha)} = \frac{\overline{A_1P_V^{AB}}}{\cos\alpha}$$

$$4\sqrt{2}R = \frac{\overline{B_1 P_V^{AB}}}{\sin(45° - \alpha)} = \frac{\overline{A_1 P_V^{AB}}}{\cos\alpha}$$

$$\overline{A_1 P_V^{AB}} = 4\sqrt{2}R\cos\alpha = 4\sqrt{2}R\,\frac{\sqrt{15}}{4} = \sqrt{30}\,R$$

$$V_{A1} = \cancel{R}\cancel{\sqrt{2}}\,\omega_{d1} = \overline{AP_V^{AB}}\,\omega_{AB1} = \cancel{\sqrt{2}}\cdot\sqrt{15}\,\omega_{AB1}\,\cancel{R}$$

$$\boxed{\omega_{AB1} = \frac{\omega_{d1}}{\sqrt{15}}}$$

$$\overline{B_1 P_V^{AB}} = 4\sqrt{2}R\sin(45° - \alpha) =$$

$$= 4\cancel{\sqrt{2}}R\,\frac{\cancel{\sqrt{2}}}{\cancel{2}}(\cos\alpha - \sin\alpha) = 4R\left(\frac{\sqrt{15}}{4} - \frac{1}{4}\right) =$$

$$= R(\sqrt{15} - 1)$$

$$V_{B1} = \overline{B_1 P_V^{AB}}\,\omega_{AB1} = R(\sqrt{15} - 1)\,\frac{\omega_{d1}}{\sqrt{15}} =$$

$$= R\left(1 - \frac{1}{\sqrt{15}}\right)\omega_{d1}$$

$$\overline{P_V^{AB}T_{1}}^2 = \overline{B_1 P_V^{AB}}^2 + (2R)^2 - 2\cdot 2R\cdot(\overline{B_1 P_V^{AB}})\cos(90° + \alpha) =$$

$$= R^2(\sqrt{15} - 1)^2 + 4R^2 + \cancel{4}R^2(\sqrt{15} - 1)\frac{1}{\cancel{4}} =$$

$$= R^2(\underline{15} - 2\sqrt{15} + \underline{1}) + \underline{4}R^2 + R^2(\sqrt{15} - 1) =$$

$$= R^2(16 + 4 - 1 - 2\sqrt{15} + \sqrt{15}) = R^2(19 - \sqrt{15})$$

$$\overline{P_V^{AB}T_1} = R\sqrt{19 - \sqrt{15}}$$

$$V_{T1} = \omega_{AB1} \cdot \overline{P_V^{AB} T_1} = \frac{\omega_{d1}}{\sqrt{15}} R \sqrt{19 - \sqrt{15}}$$

$$T_d(t_1) = 3 m R^2 \omega_{d1}^2$$

$$T_{AB}(t_1) = \frac{1}{2} m \frac{R^2 \omega_{d1}^2}{15} (19 - \sqrt{15}) +$$

$$+ \frac{1}{2} \frac{1}{12} m (4R)^2 \frac{\omega_{d1}^2}{15} =$$

$$= m R^2 \omega_{d1}^2 \left( \frac{19}{30} - \frac{\sqrt{15}}{30} + \frac{16}{360} \right) =$$

$$= m R^2 \omega_{d1}^2 \frac{61 - 3\sqrt{15}}{90}$$

$$T_B(t_1) = \frac{1}{2} m R^2 \omega_{d1}^2 \left( 1 - \frac{2}{\sqrt{15}} + \frac{1}{15} \right) =$$

$$= m R^2 \omega_{d1}^2 \frac{8 - \sqrt{15}}{15}$$

$$T_1 = \left( 3 + \frac{61 - 3\sqrt{15}}{90} + \frac{8 - \sqrt{15}}{15} \right) m R^2 \omega_{d1}^2 =$$

$$m R^2 \omega_{d1}^2 \left( \frac{90 \cdot 3 + 61 - 3\sqrt{15} + 6 \cdot 8 - 6 \cdot \sqrt{15}}{90} \right) = \frac{379 - 9\sqrt{15}}{90} m R^2 \omega_{d1}^2$$

$$\sum_{i=1}^{4} A_i = \frac{1}{2} m g R + M \frac{\pi}{2} = \frac{1}{2} m g R + 2 m g R \pi =$$

$$= m g R \left( \frac{1}{2} + 2\pi \right)$$

$$\frac{379-9\sqrt{15}}{90} m_1 R^2 \omega_{1g}^2 = m_1 g R \left(\frac{1}{2}+2\pi\right)$$

$$\omega_{1} = \sqrt{\frac{g}{R}\,\frac{45(1+4\pi)}{379-9\sqrt{15}}}$$

---

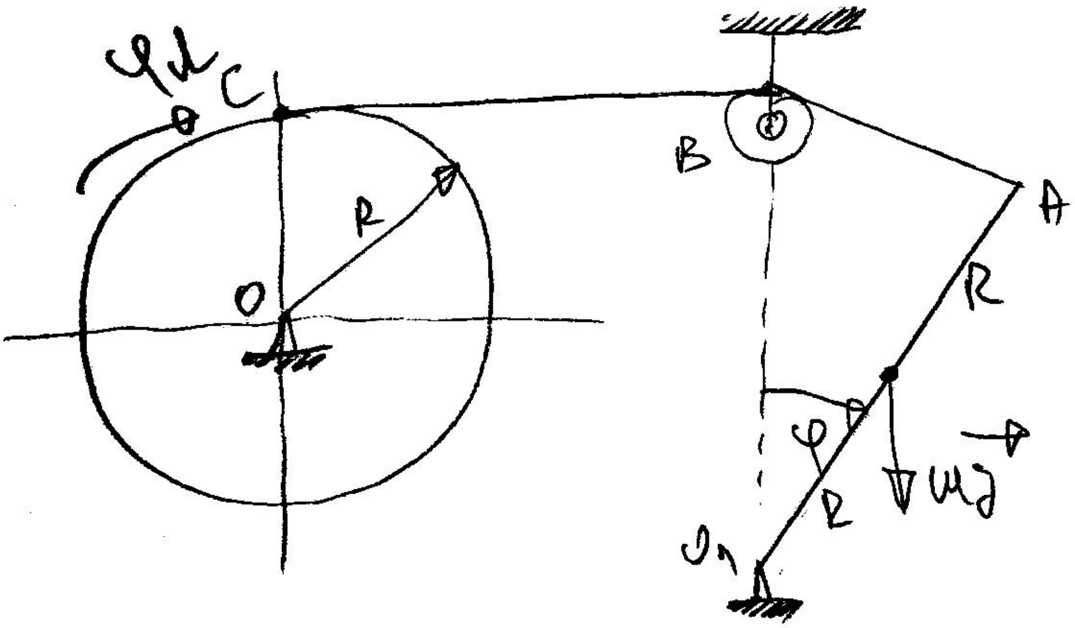
8.40 $M, R$

$O_1A: 2R, m$

$\varphi_1 = 90^\circ$

$t_0 = 0 \quad \varphi_0 = 0, \ \dot{\varphi}_0 = 0$

$\overline{O_1B} = 2R$




$t_0 = 0: \ \varphi_0 = 0, \ \dot{\varphi}_0 = 0 \qquad T_0 = 0$

$$T_1 - \underbrace{T_0}_{0} = \sum_{i=1}^{4} A_i$$

$$T_1 = \sum_{i=1}^{4} A_i = mgR$$

$R\varphi_d = S$

$S^2 = (2R)^2 + (2R)^2 - 2\cdot 2R\cdot 2R\cos\varphi =$

$= 8R^2 - 8R^2\cos\varphi = 8R^2(1-\cos\varphi)$

$S = \cancel{R}2\sqrt{2(1-\cos\varphi)} = \cancel{R}\varphi_d$

$\varphi_d = 2\sqrt{4\frac{1-\cos\varphi}{2}} = 4\sqrt{\frac{1-\cos\varphi}{2}} =$

$= 4\sin\frac{\varphi}{2}$

$\dot{\varphi}_d = 4\cdot\frac{\dot{\varphi}}{2}\cos\frac{\varphi}{2} = 2\dot{\varphi}\cos\frac{\varphi}{2}$

$v_c = R\dot{\varphi}_d = 2R\dot{\varphi}\cos\frac{\varphi}{2}$

---

$T_1 = T_{d1} + T_{OA1}$

$T_{d1} = \frac{1}{2}J_0\dot{\varphi}_d^2 = \frac{1}{\cancel{2}}\frac{1}{\cancel{2}}MR^2\cdot\cancel{4}\dot{\varphi}_1^2\cos^2\frac{90^\circ}{2}$

$= MR^2\dot{\varphi}_1^2\frac{1}{2} = \frac{1}{2}MR^2\dot{\varphi}_1^2$

$T_{OA1} = \frac{1}{2}J_{O_1}\dot{\varphi}_1^2 = \frac{1}{\cancel{2}}\frac{1}{3}m4R^2\dot{\varphi}_1^2 = \frac{2}{3}mR^2\dot{\varphi}_1^2$

$$\frac{1}{2} M R^2 \dot{\varphi}_1^2 + \frac{2}{3} m R^2 \dot{\varphi}_1^2 = m g R$$

$$\dot{\varphi}_1^2 \left( \frac{1}{2} M R + \frac{2}{3} m R \right) = m g$$

$$\dot{\varphi}_1^2 = \frac{m g}{R\left( \frac{1}{2} M + \frac{2}{3} m \right)} = \frac{6 m g R}{R(3M + 4m)}$$

$$\dot{\varphi}_1 = \sqrt{\frac{6 m g R}{R(3M + 4m)}}$$